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THE EFFECT OF GRAVITY ON THE DEPOSITION OF MICRON-SIZED PARTICLES ON SMOOTH SURFACES

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Abstract—Particle deposition on smooth surfaces from liquid suspensions involves *transport* and *attachment* steps. Transport is considered to be influenced by particle Brownian diffusivity and inertia, while attachment is the outcome of competition of hydrodynamic and physicochemical forces. In the literature, micron-size particle transport is usually modeled as a mass transfer process determined by the magnitude of Brownian diffusivity. However, even in this (colloidal particle) size range, gravity or a constant body force towards the deposition surface may significantly affect the deposition process. Image processing techniques have been used to obtain measurements of the deposition rate of micron-sized glass particles, in a horizontal narrow channel, under laminar flow conditions. Over a fairly wide range of flow rates, deposition fluxes are nearly constant. This trend, supported by a theoretical analysis, suggests that (in that range) *gravity controls* the particle transfer boundary layer thickness and the deposition rate. However, above a certain threshold flow rate, a rather sharp reduction of the deposition rate is observed indicating that the *attachment* process becomes important. The implications are discussed of the above experimental and theoretical results on modeling particulate deposition for various problems of practical interest; e.g. fouling of heat exchange surfaces or filtration membranes.

Key Words: particle deposition, sedimentation, attachment

1. INTRODUCTION

Deposition of particles from flowing suspensions is an important process in various fields of engineering and in nature. Fouling of heat transfer surfaces, deep bed and membrane filtration, atmospheric pollution, microbial and cell transport in living systems are only a few indicative areas. This paper is focused on particle deposition from liquid suspensions and the main motivation is fouling of heat transfer or filtration equipment by suspended particles.

Deposition may be considered the outcome of two consecutive steps: firstly particle transport from the bulk of a flowing suspension to the surface, and secondly attachment. An important parameter for both steps is particle size. Thus, for particles in the colloidal size range Brownian diffusivity controls transport rates, while particles of several microns in size or larger are considered to possess sufficient inertia to escape from fluid streamlines and coast to the deposition surface. Similarly, particle size is a critical parameter in determining their attachment efficiency, since it affects the magnitude of physicochemical interactions between particles and substrate, as well as the hydrodynamic forces that tend to detach or prevent them from adhering.

An account of research on particulate deposition in *laminar* flows can be found in the reviews of Jia and Williams (1990) and van de Ven (1989). Theoretical results exist for a variety of flow geometries (i.e. narrow channels and tubes, flow external to spheres and cylinders, rotating disks, etc.) investigating the effect of several parameters related to physicochemical interactions, such as van der Waals and electrostatic, as well as gravity, Brownian forces, and particle/substrate hydrodynamic interactions. In certain cases, gravity or an external body force toward the surface may determine the deposition rate. For example, Prieve and Ruckenstein (1974) analyze flow external to spheres as a model for deep bed filtration and predict that, for a range of Peclet numbers, sedimentation is the dominant mechanism. Similarly, significant effects of sedimen-

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tation are reported by Yao *et al.* (1971) for particle sizes close to one micron or larger, by Adamczyk and van de Ven (1981, 1982) for particulate deposition in rectilinear flows over flat surfaces, and by Marmur and Ruckenstein (1986) for the deposition of cells on a flat plate.

Although for laminar flows the convective environment is almost precisely known, theory and experiment are not always in satisfactory accord. For example, well known is the inability of the DLVO theory to account for observations when repulsive electrostatic forces exist (Bowen and Epstein 1979). DLVO theory cannot also account quantitatively for adhesion effects which are poorly understood. Additionally, the effects of external forces are not always clearly elucidated because they are lost in the multitude of parameters involved in the theoretical considerations. Finally, it may be added here that the amount of well-defined experimental work is small compared to the theoretical studies.

Reviews on particulate deposition in *turbulent* flows can be found in McCoy and Hanratty (1977) and Papavergos and Hedley (1984) where experimental and theoretical results are presented. A key parameter is the particle relaxation time, defined as

$$\tau_p+ = \frac{1}{18}\frac{\rho_p}{\rho}\left(\frac{u^* d}{\nu}\right)^2, \qquad [1]$$

which is a dimensionless measure of particle inertia, and reflects how closely particles follow fluid streamlines or how easily they escape from turbulent eddies. Here ρ and ρ_p are the fluid and particle densities, d is the particle diameter, ν is the fluid kinematic viscosity, and u^* is the wall shear velocity. Based on the magnitude of particle relaxation time three regimes are recognized, namely the diffusion, inertia and impaction regimes, and the following correlations are provided respectively:

$$J = u^* 0.07 Sc^{-2/3} \text{ at } \tau_p+ < 0.2$$

$$J = u^* 3.5 \times 10^{-4} (\tau_p+)^2 \text{ at } 0.2 < \tau_p+ < 20$$

$$J = u^* 0.18 \text{ at } \tau_p+ > 20. \qquad [2]$$

Here Sc is the particle Schmidt number based on the Brownian diffusivity.

Most of the experimental data, on which the above correlations are based, have been obtained with air flow. In addition, it may be pointed out that data from horizontal systems are relatively meager and display a larger scatter compared to those for vertical systems. McCoy and Hanratty (1977) suggest that when the particle settling velocity is larger than the deposition velocity corresponding to vertical systems gravity may control the deposition rate. For liquid systems gravity effects are in general less significant and the above correlations are widely used for prediction regardless of the orientation of the experimental system. However, a simple calculation indicates that the effects of gravity may still in certain cases be significant. For example, let $u^* = 10$ cm/s, $a = 1$ μm and $\Delta\rho = 1$ g/cm^3. Then the particle relaxation time is much smaller than 0.1 and deposition is diffusion dominated, according to the above model. The deposition velocity from the relevant correlation is found to be 7×10^{-5} cm/s while the settling velocity is 2.1×10^{-4}. Therefore, the effects of gravity even in liquid systems and for particles in the micron size range may not necessarily be neglected.

In the following section experimental results are reported for the deposition of micron sized glass particles on a glass substrate in a horizontal laminar flow narrow channel. Next, theoretical results are presented for laminar flow which support one aspect of the experimental findings, namely that gravity controls the transport step over a range of flow rates. Finally, some theoretical considerations for turbulent flow are presented which suggest that the effects of gravity may also, under certain conditions, be important for liquid suspensions of micron-sized particles.

2. EXPERIMENT

2.1. *Set-up and procedure*

Observations of the initial deposition rate of spherical glass particles on glass plates were conducted in a laminar flow narrow channel geometry. The test section consisted of two glass plates 10 cm long by 4.5 cm wide separated by Teflon spacer strips 0.5 mm thick, thus creating a narrow channel of 1 cm width and 0.5 mm clearance. The plate/spacer assembly was held together by a SS 316 housing which included entrance and exit sections providing smooth transition to channel flow.

The test section was placed under a microscope (Nikon Optiphot-2) equipped with transmitted and reflected light illuminators. A video camera was attached to the microscope and connected to a monitor, video-recorder and a minicomputer (Macintosh II) through a frame grabber board (Scion LG-3). In this way selected images could be digitized and analyzed for the number of deposited particles with the "NIH Image" image analysis software.

Spherical Ballotini glass particles were used to prepare the test suspensions. These were taken from a sample passed through a 325 mesh screen and fractionated in water by gravity settling to obtain a narrow size distribution. The mean size of the particles used in the experiments was 1.8 μm with a standard deviation of ~12%. Size distributions were obtained by optical measurements using the image analysis software. Particle density was determined gravimetrically by dispersing a dry sample of the initial stock in water and was found to be 2.4 g/cm^3. The fractionated particles were kept in distilled water and their concentration was determined by measuring the weight of a dried sample of known volume. Dilute suspensions of concentration approximately 2 mg/l were prepared for the deposition studies by mixing known volumes of the stock sample and the test fluid where the pH was adjusted by the addition of HNO_3.

Flow rates were measured gravimetrically and could be used to determine the hydrodynamic conditions from the known channel dimensions. The driving force for flow was hydrostatic pressure and the flow rate was regulated by a needle valve. Prior to an experiment, particle free test fluid was passed through the test section. Typical experimental duration was 30 min and images of a fixed observation area of 498×332 μm^2 were acquired for processing every 2 min. A sequence of four to eight frames was grabbed in about 1–2 s and averaged to obtain each image, in this way eliminating in focus flowing particles. Repeated runs could be performed after an *in situ* cleaning of the plates by passing distilled water at high flow rates which caused detachment of the deposited particles.

2.2. *Results and discussion*

Experiments were performed over a range of suspension flow rates at two pH values, namely pH 2 and 3. The first pH value roughly corresponds to the isoelectric point of particles and substrate and, therefore, represents conditions where electrostatic repulsive forces between particles and substrate are absent. As the pH is increased the surfaces become negatively charged, and therefore electrostatic forces should have a progressively larger effect, eventually preventing any

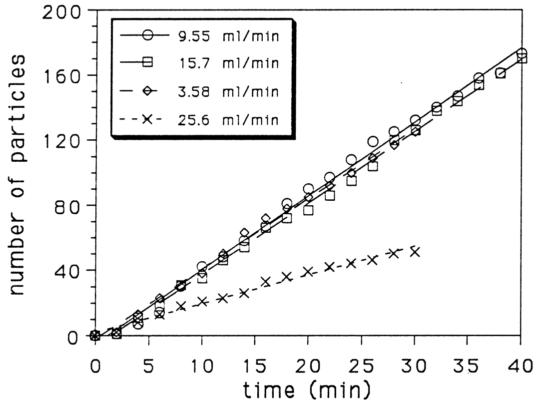

Figure 1. Sample experimental results for deposition at pH 2.

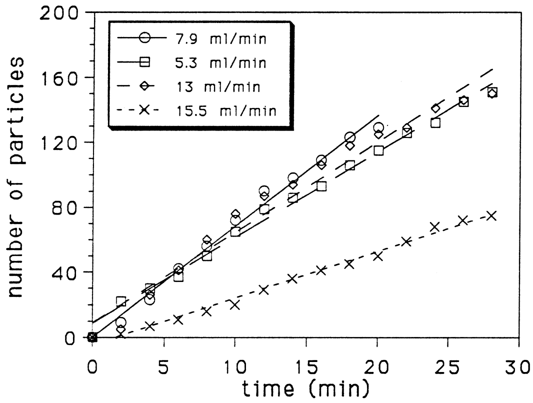

Figure 2. Sample experimental results for deposition at pH 3.

deposition. At pH 3 the ζ-potentials are in the range of −10 to −20 mV (Yiantsios and Karabelas 1995).

Typical data from the experimental runs at pH 2 and 3 are shown in figures 1 and 2. From the known size of the observation area, the bulk suspension concentration and the particle size and density, the slope of the deposition curves similar to those in figures 1 and 2 can be used to obtain the deposition rate as a function of the hydrodynamic conditions near the deposition surface. Such results are presented in figures 3 and 4 where the deposition rate is plotted as a function of the hydrodynamic shear stress at the plate surface.

It is observed that the deposition rates, up to a certain threshold shear stress, are constant and identical for the two pH values to within experimental error. Superimposed on the diagrams is also a straight line representing the settling velocity of the particles, as calculated from their properties (size, density and the viscosity of water). Evidently, there is satisfactory agreement between the measured deposition rates and the settling velocity, and this suggests that gravity controls the deposition process over a range of relatively low shear stresses. In support of this conclusion it may be added here that no deposition at all took place on the upper plate of the channel under any conditions, as was observed by examination of the plate after the completion of each experimental run. With gravity pointing away, particles are effectively prevented from being transported to the upper plate.

Above a certain threshold shear stress the deposition rates become significantly lower. This behavior, which is contrary to expectations from convective transport considerations, suggests that the attachment step tends to play a role. More precisely, deposition becomes affected by the competition between adhesive and hydrodynamic forces on the particles. If the latter become sufficiently large they prevent particles from adhering to the substrate and can also detach predeposited particles. It must be noted here that the observed shear stresses at which hydrodyn-

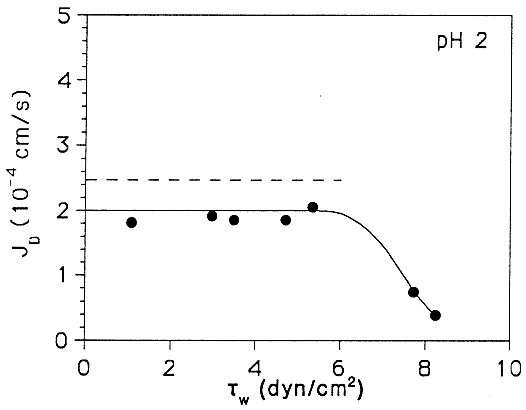

Figure 3. Deposition rate as a function of the hydrodynamic wall shear stress at pH 2. The dotted straight line represents the free settling velocity.

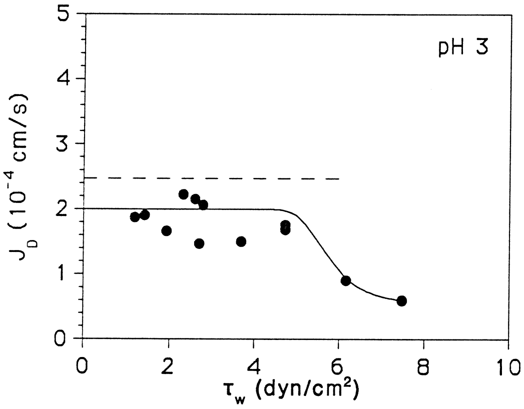

Figure 4. Deposition rate as a function of the hydrodynamic wall shear stress at pH 3.

amic forces begin to compete with adhesive forces are relatively low compared to those measured in detachment experiments, previously performed in the same apparatus (Yiantsios and Karabelas 1995). Although these differences cannot be explained quantitatively, they are hardly unexpected, because the time particles interact with the substrate is vastly different in the two cases. In the detachment experiments particles are predeposited under quiescent conditions and are allowed to rest on the substrate for at least 1 h, whereas in the present experiments particles failing to adhere interact with the substrate only momentarily.

One may argue that a different explanation for the observed behavior exists, namely that the reduction of the deposition rates is due to the competition between gravity and inertial lift forces acting on the particles. The latter are often mistakenly identified with the Saffman lift forces which pertain to particles exposed to unbounded shear flows (see also the discussion in Yiantsios and Karabelas 1995). The inertial lift forces, appropriate for particles in the vicinity of the deposition surface as in the present case, are given by the analysis of Leighton and Acrivos (1985) who find

$$F_L = 0.58\rho(\tau_w/\mu)^2 d^4. \qquad [3]$$

Here ρ and μ are the fluid density and viscosity, τ_w is the wall shear stress and d is the particle diameter. It is easy to verify, however, that the inertial lift forces are at least an order of magnitude smaller than the gravity force on the particles for all the conditions of the present experiments, and therefore do not play any role.

Finally, it is worth observing that, although at pH 3 electrostatic forces should be important according to DLVO theory calculations (Yiantsios and Karabelas 1995), no significant effect thereof is apparent, apart from a rather small reduction of the threshold shear stress above which the attachment process prevails. This is reminiscent of the well-known discrepancy between DLVO theory calculations and similar experiments (Bowen and Epstein 1979), where theory predicts that no deposition at all should take place while measurable deposition rates are obtained experimentally.

3. THEORY

3.1. *Laminar flow*

It is fairly straightforward to show by theoretical transport considerations that gravity determines the deposition rates when the attachment process is inactive, i.e. when hydrodynamic forces on the particles are too weak to compete with adhesion forces. The simplest continuum description of particle transport is one which neglects particle/substrate hydrodynamic interactions, physicochemical interactions and any adhesion effects. This is given by

$$\gamma y \frac{\partial c}{\partial x} - U_G \frac{\partial c}{\partial y} = \mathscr{D} \frac{\partial^2 c}{\partial y^2}, \qquad [4a]$$

where x is the longitudinal coordinate, y the vertical with origin at the wall, γ the shear rate, U_{G} is the particle free settling velocity, and $\mathscr{D}$ the particle Brownian diffusivity. Here, streamwise diffusion effects are neglected, being small compared to convective effects, and the velocity profile is approximated by linear shear assuming that the thickness of the concentration boundary layer is small compared to channel dimensions. Initial and boundary conditions need to be specified to complete the problem as follows:

$$c = c_{\mathrm{b}} \text{ at } x = 0$$

$$c \longrightarrow c_{\mathrm{b}} \text{ as } y \longrightarrow \infty$$

$$c = 0 \text{ at } y = 0, \qquad [4b]$$

where the latter implies perfect adhesion of the particles that reach the wall.

It is evident that [4] possesses an asymptotic solution, independent of x, which is determined by the balance of sedimentation and diffusion effects, provided that $U_{\mathrm{G}} > 0$:

$$c_{\infty} = c_{\mathrm{b}}(1 - e^{-U_G y/\mathscr{D}}). \qquad [5]$$

Accordingly, the deposition flux, after a transition length, becomes

$$J = U_{\mathrm{G}} c_{\mathrm{b}} \qquad [6]$$

and the deposition rate becomes equal to the particle settling velocity, as observed experimentally.

The only question remaining then is what is the transition length and whether this is significant compared to channel dimensions. Evidently, if U_{G} is sufficiently small this length will be unrealistically long and the effects of sedimentation will be practically insignificant. Furthermore, the assumption of a small boundary layer thickness compared to channel dimensions will be also invalid. Equation [4a] can be put in a dimensionless parameter-free form by scaling c with c_{b}, y by $\mathscr{D}/U_{\mathrm{G}}$, and x by $\gamma\mathscr{D}^2/U_{\mathrm{G}}^3$, to yield

$$y\frac{\partial c}{\partial x} - \frac{\partial c}{\partial y} = \frac{\partial^2 c}{\partial y^2}. \qquad [7]$$

Therefore, the transition length, according to this simplified model is of the order of $\gamma\mathscr{D}^2/U_{\mathrm{G}}^3$.

It is interesting to investigate further whether the conclusions of this simplified approach still hold when some of the assumptions are relaxed. In particular, it is interesting to consider the effects of hydrodynamic interactions, since it is known that the particle vertical mobility is reduced at close approach to the wall, as is also the case with the vertical component of diffusivity. Both become proportional to separation at small distances. Thus, hydrodynamic interactions would prevent particles from coming to contact with the substrate in the absence of attractive physicochemical interactions. In many modeling efforts the assumption is used that the two effects cancel each other, which would justify the simplified approach presented above. However, while attractive van der Waals forces act over separations of the order of a few nanometers, hydrodynamic interactions are important at separations of a few particle diameters. Thus, their length scale is different for particles in the micron size range and their effects may be important when the concentration boundary layer thickness is of the order of particle size. In this case the precise form of the attractive interaction potential is not important and the effect of attractive forces may be simply accounted for by assuming that particles are irreversibly captured when they arrive at a finite distance from the wall. The precise magnitude of this distance is also not important as long as it is much smaller than the concentration boundary layer thickness.

Given the above, a continuum description of particle transport takes the form

$$\gamma F_1 y\frac{\partial c}{\partial x} - U_{\mathrm{G}}\frac{\partial F_3 c}{\partial y} = \mathscr{D}\frac{\partial}{\partial y}\left(F_3\frac{\partial c}{\partial y}\right), \qquad [8]$$

where F_1 and F_3 are universal hydrodynamic correction factors for the tangential and vertical mobility of particles near a wall relative to those in unbounded fluid (Goldman *et al.* 1967; Goren and O'Neill 1971). Equation [8] can be put in dimensionless form by scaling y by the particle radius a, x by $\gamma a^2/U_G$, and c by c_b

$$F_1 y \frac{\partial c}{\partial x} - \frac{\partial F_3 c}{\partial y} = \frac{1}{N_G}\frac{\partial}{\partial y}\left(F_3 \frac{\partial c}{\partial y}\right), \qquad [9a]$$

with initial and boundary conditions

$$c = 1 \text{ at } x = 0$$

$$c \longrightarrow 1 \text{ as } y \longrightarrow \infty$$

$$c = 0 \text{ at } y = \delta. \qquad [9b]$$

The parameter N_G is defined as $N_G = 4\pi\Delta\rho g a^4/3kT$ and reflects the relative magnitude of gravity and Brownian forces. The parameter δ reflects the effects of attractive forces. As explained in the previous paragraph, δ needs to be assigned a finite value since particle mobility vanishes at $y = 0$, but the exact value is unimportant as long as it remains much smaller than other macroscopic transport length scales (see also Adamczyk and van de Ven 1981).

It may be observed that again an asymptotic state, independent of x, exists. Neglecting the transient terms in [9a] and integrating once one obtains

$$F_3(y)\left(c + \frac{1}{N_G}\frac{\partial c}{\partial y}\right) = \text{const} \qquad [10]$$

and since for $y \gg 1$, $c \rightarrow 1$, $F_3 \rightarrow 1$, $\partial c/\partial y \rightarrow 1$ the constant is equal to 1. The same relation applied at $y = \delta$, where $c = 0$, can be used to give the deposition rate, which in dimensional form is again found to be

$$J = U_G c_b, \qquad [11]$$

as is also given by [6]. It can be concluded, therefore, that the asymptotic deposition rate is not affected by hydrodynamic effects on particle mobility, and is still equal to the particle free settling velocity.

However, hydrodynamic interactions have other important effects. The asymptotic concentration profile is different from that given by [5]. Thus, [10] integrated once more gives

$$c = N_G e^{-N_G y} \int_\delta y e^{N_G y} F_3^{-1} \, dy. \qquad [12]$$

This is shown in figure 5 for $N_G = 1$. An increased concentration over that in the bulk exists near the surface, which compensates for the decreased particle mobility. The maximum is of the order of N_G and occurs at a dimensionless distance of the order of $1/N_G$ from the surface. Such a behavior is reminiscent of concentration polarization in membrane filtration where the role of gravity is played by drag on the particles due to permeation flow.

A second effect of hydrodynamic forces is on the transition length required to reach the asymptotic profile and deposition rate. In order to find this effect [9] was solved numerically using finite differences. The dimensionless length required for the deposition rate to reach the asymptotic value to within 10% is plotted in figure 6 as a function of the dimensionless parameter N_G. As expected, for small N_G this length approaches the value given by the analysis neglecting hydrodynamic corrections, since, in this case, the concentration boundary layer thickness is much larger than particle size. However, for larger values of N_G the approach is slower, and for N_G larger than 0.2 the dimensionless length is approximately 60. In dimensional terms this is equivalent to a value independent of particle size, given by $L \sim 250\,\tau_w/\Delta\rho g$. For the present experiments, where the hydrodynamic shear stress was less than 10 dyn/cm^2, the transition length is less than 2 cm. Therefore, the distance of the observation area from channel entry dis-

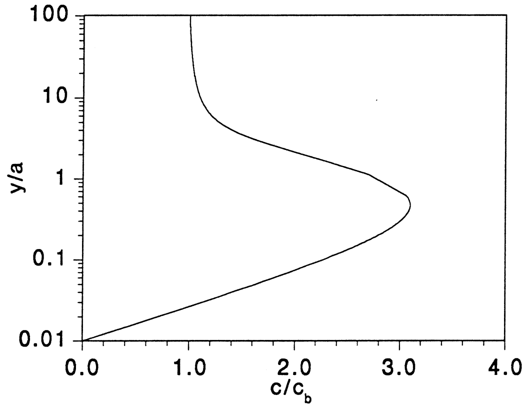

Figure 5. The particle concentration profile when hydrodynamic mobility corrections are included for $N_G = 1$.

tance, which in the present experiments was 5 cm, was sufficient for the asymptotic state to be reached. For filtration membranes, where gravity is replaced by the more significant permeation drag, this length will usually be small compared to practical equipment dimensions.

3.2. *Considerations for turbulent flows*

Some preliminary considerations for particle deposition in turbulent flows are finally included here in order to demonstrate that sedimentation effects may also be important in turbulent flows of liquid suspensions of micron-sized particles. McCoy and Hanratty (1977) suggest that if the settling velocity of aerosols is higher than deposition rates for vertical systems, gravity plays a significant role. A similar conclusion is drawn by the theoretical analysis of Dabros and van de Ven (1983), where it is assumed that particle transport occurs mainly by velocity fluctuations due to downsweeps and bursts, which are modeled as steady two-dimensional stagnation flows.

Several surface renewal models have been employed to analyze mass transfer under turbulent flow conditions (Ruckenstein 1987). A similar approach is presented below which is largely based on a model for molecular mass transfer in turbulent flows by Campbell and Hanratty (1983). It is assumed that transverse velocity fluctuations near the wall comprise the dominant convective mechanism while streamwise fluctuations being of a larger lengthscale and lower energy are neglected. Since the Schmidt numbers are large the concentration boundary layer thickness is small compared to the viscous sublayer thickness, and, therefore, only the limiting behavior of velocity field is sufficient. Hence, the vertical and spanwise fluctuations are taken quadratic and linear in the vertical coordinate, respectively. It is further assumed that the velocity field has the form of counter-rotating eddies, fixed in space and separated by a distance λ of approximately 100 wall units. Then, the vertical and spanwise velocity components can be found from continuity to be

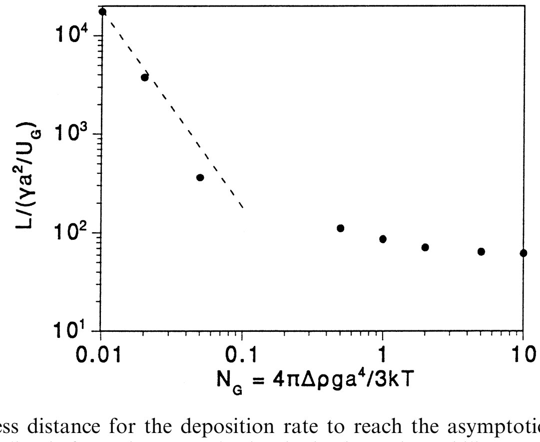

Figure 6. Dimensionless distance for the deposition rate to reach the asymptotic value to within 10%. Dotted line is from theory neglecting hydrodynamic mobility corrections.

$$w = -\beta(t)\cos(y/\lambda)z^2/2;\ v = \beta(t)\sin(y/\lambda)z. \quad [13]$$

Scaling z and t by wall units and the spanwise coordinate λ one obtains

$$\frac{\partial c}{\partial t} + \beta(t)\left[F_1 z\frac{\partial c \sin y}{\partial y} - \frac{\cos y}{2}\frac{\partial z^2 F_2 F_3 c}{\partial z}\right] - \frac{U_G \partial F_3 c}{u^* \partial z} = Sc^{-1}\frac{\partial}{\partial z}\left(F_3\frac{\partial c}{\partial z}\right), \quad [14]$$

where Sc is the Schmidt number and F_1, F_2, F_3 are hydrodynamic mobility corrections. In contrast to Dabros and van de Ven (1983), these corrections are included here because for micron sized particles their length scale is larger than that of van der Waals forces. Finally, if the vertical coordinate is rescaled by $(<\beta>Sc)^{1/3}$, and time by $<\beta>^{2/3}Sc^{-1/3}$, where $<\beta>$ is the mean square value of velocity fluctuations, the transport equation takes the form

$$\frac{\partial c}{\partial t} + \beta(t)\left[F_1 z\frac{\partial c \sin y}{\partial y} - \frac{\cos y}{2}\frac{\partial z^2 F_2 F_3 c}{\partial z}\right] - N_{G^*}\frac{\partial F_3 c}{\partial z} = \frac{\partial}{\partial z}\left(F_3\frac{\partial c}{\partial z}\right), \quad [15]$$

where

$$N_{G^*} = U_G/u^* <\beta>^{1/3} Sc^{-2/3} \quad [16]$$

The parameter N_G* is a measure of sedimentation relative to the deposition rate due to convective diffusion.

With the scaling used here, if sedimentation is absent and hydrodynamic corrections are neglected, the only parameter that remains is the timescale of the velocity fluctuations, appearing implicitly in the forcing function $\beta(t)$. Furthermore, this scaling yields immediately the correct functional dependence of the deposition rate since the mass transfer coefficient $k = O(\mathscr{D}z_0/c_b) = O(u^* < \beta >^{1/3}/Sc^{2/3})$. It must also be noted that the time scale of mass transfer fluctuations, calculated for indicative values of $Sc = 10^6$ and $<\beta> = 0.005$ (Finnicum and Hanratty 1985), is of the order of 3000 wall units. This suggests that only low frequency velocity fluctuations are important in mass transfer and, furthermore, that the assumption of a quasi-steady velocity profile employed in Dabros and van de Ven (1983) may not necessarily be correct. Admittedly, however, unsteady models, apart from appearing more realistic, do not always lead to significant improvements from a practical point of view (Ruckenstein 1987).

Campbell and Hanratty (1983) used an experimentally measured signal for $\beta(t)$ in their numerical calculations. However, since only low frequencies are important they state that any white noise signal would also be sufficient. In the preliminary calculations presented here a white noise signal was used for $\beta(t)$ with a low frequency cut-off at 0.1 of the mass transfer time scale. This is close to the experimental signal given by Campbell and Hanratty. The transport equation was solved by finite difference techniques. In these numerical calculations an implicit scheme was used for time advancement and the convective terms were approximated by semi-implicit upwind differencing. A non-uniform grid was used to capture the rapid concentration variations near the deposition surface. The time and space averaged deposition rate as a function of the parameter N_G* is shown in figure 7. It may be observed that the model predictions for zero gravity are close to mass transfer correlations, while a significant effect of sedimentation is also apparent when N_G* is different from zero.

Some calculations were also performed taking into account hydrodynamic mobility corrections. In this case, however, additional parameters appear explicitly, namely the ratio of particle size to concentration boundary layer thickness and the capture distance compared to boundary layer thickness. Given the preliminary character of such calculations a thorough study was not performed. Only an indicative calculation is presented here where the two parameters are set to 1 and 0.01, respectively. These values roughly correspond to micron sized particles and shear velocities of the order of 10 cm/s. The results are also shown in figure 7 where, again, a significant effect of sedimentation is predicted since the values of the parameter N_G* studied can be easily realized with non-neutrally buoyant micron-sized particles.

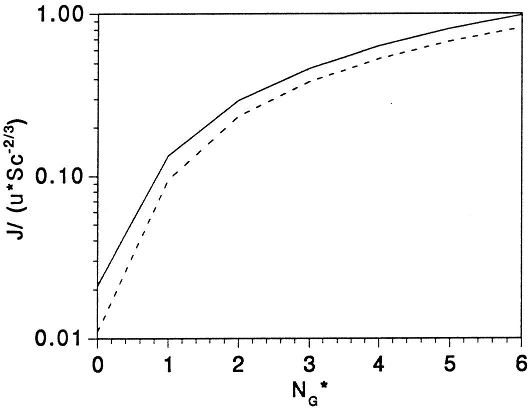

Figure 7. Deposition rate as a function of the parameter N_G*, according to the counter-rotating eddies model for near-wall turbulence. Dotted line represents calculations including hydrodynamic mobility corrections.

4. CONCLUDING REMARKS

Experimental data obtained under well defined hydrodynamic and physicochemical conditions suggest that in horizontal laminar parallel flows gravity controls the deposition rate of non-neutrally bouyant micron-sized particles over a range of hydrodynamic wall shear stresses. A theoretical analysis taking into account finite particle size effects confirms this conclusion, as well as that entrance effects may not be practically significant. It further suggests that these effects may also be insignificant when the role of gravity is taken up by the more important permeation drag in cross-flow filtration of colloidal particles. Particle/substrate hydrodynamic interactions are also shown to have another important effect which is an increase of particle concentration near the substrate over the bulk value. This may reconcile the approaches taken in modeling deposition on impermeable and permeable surfaces where in the former surface concentration is taken to be zero while in the latter it is taken to be the gelation concentration due to the concentration polarization effect.

When hydrodynamic shear stresses exceed a certain threshold, the deposition rates appear to be influenced by the attachment process, involving a competition between adhesive and hydrodynamic forces. It is, therefore, suggested that measurements under such conditions, where transport to the substrate can be easily and accurately estimated, may be very useful for studying the particle sticking probability which is essential for reliable deposition rate predictions, but at present is poorly understood. Furthermore, additional work is obviously needed to determine the threshold shear stress or criteria for transition from a transport controlled to an attachment dominated deposition process.

Simple arguments and preliminary theoretical considerations suggest that sedimentation effects may be also important in turbulent flows of liquid suspensions of micron-sized particles in horizontal arrangements. Such effects require careful experiments, given also the influence of adhesion, and at present cannot be definitely detected from particulate fouling studies where usually only macroscopic fouling resistance measurements are reported in the relevant literature.

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